**An Approach to Solving Heteroskedacity when the variance is a Multiplicative Constant**

Consider a scale that measures two objects. For each object, we have some randomized error. Consider the weights of two objects, first alone and then together.

That is, consider the following model:

This is like a regression model, in that we can solve for the alphas (the true weights) by the method of least squares.

Let , , and where y is the measured value, X is the design matrix, α is the true value and ε is the random error (a multivariate normal variable with mean 0 and covariance matrix I. ) Then we can model the response as follows   
  
To predict α, we will use the least squares approach. By minimizing the least squared error, we know that the prediction for α is:

Solving this,

so,

Now consider that the random element is not equally variant for the two objects – that is, that for both weights combined there is some multiplicative factor on the error > 1. That is, )**.** Note that this violates the least squares assumptions. One method to fix this issue is to transform the variables so they have equal variance, since k is a numeric value > 0.

Note that we can transform the third variable as follows:

Now, has variance .

Let , , and

However, this is not a general solution. Consider instead the following:

)**.**

We could attempt to perform the same procedure, that is, divide by , by and so on. However, this is arduous, and it would be convenient to use a matrix of weights to solve this.

Let ***W*** be a matrix of the reciprocals of these constants. Using the same procedure as above, we can express as  and as **.** Then our result is:

We can see that if **W** = **,** we get the same result as earlier.